Chapter 4: Permutations

In the latter part of the 12th century Arnaut Daniel, a troubadour from Ribérac in what is now the Dordogne, entertained the courts of southern Europe with poems on themes of chivalry and courtly love. Daniel's poetry, written in his native Occitan, is characterised by technical virtuosity, with complex rhyme and metrical schemes and intricate structures. Although only a few of his poems are still extant, his gifts have impressed successive generations of poets: Dante, Petrarch and Ezra Pound all held him in the highest regard.

Daniel is generally credited with inventing the sestina and his poem 'Lo ferm voler qu'el cor m'intra', dating from around 1200, is the earliest known example of this poetic form. A sestina consists of six stanzas of six lines each, with each stanza featuring the same end-words in a set sequence of permutations. In addition, a traditional sestina concludes with a three-line envoi that contains some or all of these end-words. If we denote the end-words by the letters A, B, C, D, E, F, then they rotate through the stanzas in the following sequence:

Line	Stanza 1	Stanza 2	Stanza 3	Stanza 4	Stanza 5	Stanza 6
1	A	F	C	E	D	B
2	B	A	F	C	E	D
3	C	E	D	B	A	F
4	D	B	A	F	C	E
5	E	D	B	A	F	C
6	F	C	E	D	B	A

You will see from this scheme that the order of the end-words is unique to each stanza. This is a fundamental characteristic of permutations: order matters. (As an illustration, the words *apt*, *pat* and *tap* are all permutations of the letters *a*, *p*, and *t*.)

The tight constraints make sestinas challenging but also fun to write. Dante and Petrarch both composed sestinas. Over the centuries numerous poets have taken on this technically demanding form, including writers as diverse as Sir Philip Sidney, Rudyard Kipling, Seamus Heaney and Elizabeth Bishop. The form has given rise to many adaptations, such as Swinburne's rhyming double sestina 'The Complaint of Lisa' or Marvin Thompson's 'Triptych', in which each of the nine stanzas has seven lines. 'Six Words' by Lloyd Schwartz is a delightfully playful sestina consisting entirely of permutations of the words *yes*, *no*, *maybe*, *sometimes*, *always* and *never*, made meaningful by adroit use of font variation and punctuation.

American poet Marie Ponsot invented a condensed form called the tritina, which she described, rather oddly, as 'the square root of the sestina'. A tritina consists of three tercets with three end-words in the sequence 123, 312, 213, and a single final line containing all three end-words.

This leads to the question: for what numbers *m* is an *m*-tina possible? This was first investigated in the 1960s by the French poet Raymond Queneau, co-founder of the Oulipo movement, and his mathematician colleague Jacques Roubaud, and has continued to be a fruitful topic of mathematical research. It turns out that 9-tinas and 11-tinas are possible, but 10-tinas are not. And who would want to read — let alone write! — a 194-tina?

Queneau was fascinated by the creative possibilities of using mathematics in his writing. In 1961 he published 'Cent mille milliards de poémes' ('A Hundred Thousand Billion Poems'), a sequence of ten sonnets constructed in such a way that any given line from one sonnet can be interchanged at

random with the corresponding line from any of the other sonnets. There are therefore 10 possibilities for the first line, 10 for the second line and so on, generating a total of 10^{14} distinct sonnets. No one has ever read all the permutations in their entirety: by Queneau's own estimate it would take 190,258,751 years, reading night and day without a break, to do so.

Francois Le Lionnais, co-founder with Queneau of the Oulipo movement, devised the concept of a multi-choice narrative, in which the reader can choose a route through a story or poem. Queaneau plays with this non-linear structure in his light-hearted 'A Tale for your Shaping'. In the opening line the poet asks:

> 1 — Would you like to know the story of the three lively little peas?

If our answer is no, we are then invited to hear about 'the three tall thin beanpoles' instead; or, failing that, 'the three middle-sized middling bushes'. At the end of each numbered section we are offered further choices, so the narrative darts around crazily until the various threads are drawn together in the last two sections.

The lipogram is another form of constrained writing that was taken up by the Oulipo movement, although its roots go back to ancient Greece. The word 'lipogram' is derived from the Greek λειπογράμματος (leipogrammatos), meaning 'to leave out a letter', and describes a piece of writing in which certain letters of the alphabet are omitted. Lasus of Hermione, a Greek lyric poet and musician who lived during the 6th century BC, wrote at least two pieces omitting the letter sigma (s), perhaps because he did not like how it sounded when sung.

The SATOR square, which we considered in the previous chapter, is a lipogram: each line is a permutation of some of the 8 letters *a, e, n, o, p, r, s, t*.

Contemporary poets have explored the creative possibilities of the lipogram as a device that can either liberate or constrain. My poem 'Scylla and Charybdis' uses lipogrammatic restrictions (the 11 letters of the title) to emphasize a sense of entrapment:

Scylla and Charybdis

Scylla

Charybdis

a hiss, a snarl, a cry.
scaly hybrid hydra
hid in
a rancid lair
rabidly
raids all.

hail. rain.
birds chirr
in briny air.
chill abyss
birls daily;
slays all.

sail by, inch by inch.
dry land calls.

By contrast, Luke Bradford has utilized the space within lipogrammatic constraints to generate dynamic, vivid word-pictures. His book *Zoolalia* is a collection of lipograms formed from the letters that make up the Latin names of various animal species. The results are lyrical and evocative, as in his poem below.

ARCTIC HARE

Lepus arcticus

a restless, alert spirit,
it traces a trail,
races past a pastel icescape as pale as erasure

In these three lines, containing only 10 distinct letters, Bradford finds space to convey the energy of the hare, its presence within the Arctic landscape and its speed and lightness of movement.

A special case of the lipogram is the anagram, in which all the letters of a word or phrase are rearranged to form a different word or phrase. The Oulipian poet Michelle Grangaud has invented a daunting poetic form known as the sestanagrammatina, a combination of the sestina and the anagram. Each line in her sestina 'Le grand incendie de Londres' is an anagram of the poem's title.

Anthony Etherin has written a number of anagram poems (including a sestanagrammatina for Pablo Picasso). Here's his aptly named 'Permutations', with each line an anagram of the poem's title:

Permutations

Atoms erupt in
mutant prose. I
turn a poem – its
matter is upon
me, to trap us in
utopian terms….

At resumption,
I must open art,
or input a stem
torn up as time –
use important
permutations.

In *The Utu Sonnets* Etherin has pushed the poetic possibilities of the anagram even further. The book consists of a sequence of seven sonnets, each of which – in addition to other constraints – is an exact anagram of all the others.

Howard Bergerson, editor of *Word Ways* – an American journal devoted to recreational linguistics – invented the vocabularyclept poem, formed by rearranging the words of an existing poem. In 1969 he challenged the journal's readers to construct a new poem using the vocabulary of his own (24 lines long) 'Winter Retrospect'. A solution was submitted by the English poet, puzzle enthusiast and gifted recreational mathematician J. A. Lindon who, astonishingly, gave his version the same title even though he had not referred to Bergerson's original. Lindon's account of how he went about finding his solution is an object lesson in dogged persistence. Self-deprecatingly, Lindon observes that the result of his endeavours 'does make some sort of sense and, like genuine poetry, does seem to have elusive undercurrents of meaning.'

In the 1950s the innovative artist, writer and sound poet Brion Gysin experimented with what is known as the cut-up technique — cutting up and rearranging pieces of text. This led him to write a series of permutation poems, each consisting of permutations of the words in the title. Together with the mathematician Ian Sommerville he pioneered the use of computer programming to generate

permutated text: an early example of the application of technology in poetry. Gysin's most famous permutation poem, which has several versions, is entitled 'I AM THAT I AM'.

Many of Gysin's books were published by the London-based Writer's Forum, established by Bob Cobbing. Cobbing's concrete 'Square Poem' (featured in Chapter 2) also uses permutations in its structure.

Educator, poet and polymath Susan Gerofsky has developed multi-sensory, inter-disciplinary methods to build robust understanding of mathematical concepts. To explore permutation patterns, she intertwines poetry, plaiting bread dough and bellringing (all of which sounds much more fun than learning about permutations while seated behind a desk in a stuffy classroom!).

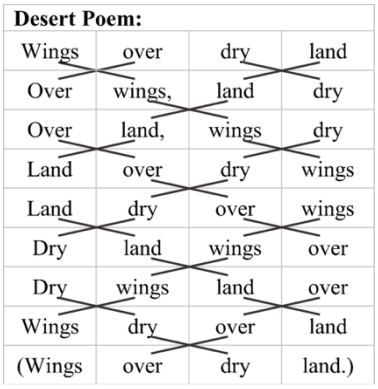
The practice of ringing a set of tuned bells in a given sequence is known as change-ringing and originated in 17th century England. Gerofsky's 'Desert Poem' is structured according to a change-ringing sequence on four bells called the Plain Hunt on 4 (mathematically speaking, this is a subgroup of the permutations on a set of four objects).

Desert Poem

Wings over dry land
Over wings, land dry
Over land, wings dry
Land over dry wings
Land dry over wings
Dry land wings over
Dry wings land over
Wings dry over land --
Wings over dry land.

The diagrams below show the Plain Hunt on 4 sequence and how it relates to the poem. The numbers 1, 2, 3 and 4 represent the four bells.

Plain Hunt on 4			
1	2	3	4
2	1	4	3
2	4	1	3
4	2	3	1
4	3	2	1
3	4	1	2
3	1	4	2
1	3	2	4
1	2	3	4



Reading the poem aloud, we can hear how the words resonate in harmony with each other.

Stephanie Strickland's innovative collection *Ringing the Changes* also draws on bell-ringing traditions, taking as its starting point permutations of ringing a set of seven bells in sequence. In place of bells, however, Strickland has used shareable code to generate text. As she notes in her Afterword, 'Sounding from a bell tower changes are samples of sound, but in this book they are samples of

language.' The effect is an intricate weaving of juxtaposed texts from a range of sources, inviting the reader to reflect on change, shifting perspectives and interconnectivity in a dynamic context.

Strickland gives us a sense of this approach in the digital project *Liberty Ring!* which she describes as 'a toy interactive companion to Ringing the Changes'. With each 'sounding' of the bell (by clicking on it with the mouse cursor) a set of seven statements appears on the computer screen for us to read and contemplate, in an experience that is both playful and profound.

Permutations feature in many other standard poetic forms not considered here: in rhyme and metrical patterns, in the structure of the villanelle and pantoum, as well as in German and Russian combinatorial literature. From ancient traditions and mediaeval troubadour songs to contemporary code-generated text, permutations and poetry interweave in creative works of dazzling virtuosity.

Further reading

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