Chapter 6: Number sequences

Sometime around 1550 BC an Egyptian scribe named Ahmes noted down a method for obtaining the area of a circle, in what is the earliest recorded attempt to evaluate the number we know as π, that is, the ratio of a circle's circumference to its diameter. The history of π (its symbol is the Greek letter pi) is fascinating, as are the many mathematical formulae for determining its value. It features in what is widely regarded as mathematics' most beautiful expression, a perfect poem in itself, Euler's identity

$$e^{i\pi} + 1 = 0.$$

Both π and e, the exponential constant – also known as Euler's number and discussed later in this chapter – are irrational numbers: they have infinite, non-recurring decimal expansions. (The other symbol in this expression, i, represents the complex number $\sqrt{-1}$, which has the characteristic that $i^2 = -1$.) To 16 digits, the expansion of π is

$$\pi = 3.141592653589793.$$

In pre-calculator days, mnemonics known as 'piems' (pi-poems) were sometimes used to commit to memory the value of π to a given number of digits. One I could relate to as an undergraduate is attributed to the mathematician and astronomer James Jeans:

How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics!

Jacques Bens, a founding member of the Oulipo movement, invented the *sonnet irrationnel* – irrational sonnet – which has five stanzas of 3, 1, 4, 1 and 5 lines respectively (the first five digits of π, adding up to a total of 14 lines), and also conforms to specific rhyme and metrical patterns.

A recent variation of the piem is the piku, of which there are several versions. One is a three-line poem with 3 syllables in the first line, 1 in the second and 4 syllables in the third line. Another version is a haiku with lines of 5, 7 and 5 syllables respectively and the additional constraint that the number of letters in each word corresponds to the equivalent digit in the expansion of π.

Here's an off-the-cuff piku of my own:

Sun. I find a beach,
windswept, to paddle among
the foamy wavelets.

In principle there is no limit to the length of a poem or piece of prose constrained in this way by successive digits of π. Problems arise, however, when the digit is zero, or when there are several low digits in succession, for example 1121. To overcome such issues a set of rules has been developed, known as Pilish, which the American mathematician Michael Keith explains as follows:

In Standard Pilish**,** each word of n letters represents

(1) The digit n if $n < 10$

(2) The digit 0 if $n = 10$

(3) Two consecutive digits if $n > 10$
(for example, a 12-letter word represents the digits 1,2).

Keith has produced several remarkable works of constrained writing in Pilish, including a novel, a retelling of Edgar Allan Poe's 'The Raven', and a circle poem using the first 402 digits of π.

He has also created his own elegant version of the piku, written in Pilish with 3, 14 and 15 syllables per line respectively:

> It's a moon,
> A wheel revolving on golden earth, and lotus blossoms.
> Mountains embrace windmills, and it all reflects this number, pi.

Jostling with π for top place in the pantheon of irrational numbers is *e*, the exponential number. Its discovery is generally credited to Jacob Bernoulli, a Swiss mathematician who in 1683 investigated a problem about compound interest that required him to evaluate the following expression as *n* becomes very large:

$$\left(1+\frac{1}{n}\right)^n$$

The approximate value is 2.71828..., which we now denote as *e*. Among its many applications, *e* forms the base for natural logarithms and features in exponential growth and decay. In her delightful poem 'The Enigmatic Number *e*', Sarah Glaz takes us on a whistle-stop tour of *e*'s history, noting in conclusion:

> And now and then *e* stars in published poetry—
> honors and administrative duties multiply with age.

The number *e* certainly plays a starring role in Anthony Etherin's 'Asymptote'. The poem is an aelindrome, an extension of the palindrome invented by Etherin himself. In this form words are divided according to a specific numerical sequence into letter units, that are then reversed around a pivot. 'Asymptote' is structured around the expansion of *e* to 20 digits: *e* = 2.7182818284590452353….

Asymptote

Infinite sets,
to unity,
present critical
enterprises,
over a map.

Ether, I eat ash.

To me,
asymptotic log is key,
logistic to a symptom,
eerie, at a shape ….

The ramp-rise
(so vertical, entire)
sent crypts
to unite
finites in.

The first four letter-units, corresponding to the first four digits in the expansion of *e*, are: $[\text{In}]_2$ $[\text{finite s}]_7$ $[\text{e}]_1$ and $[\text{ts to unit-}]_8$. They reflect backward from the last line of the final stanza as $[\text{in}]_2$ $[\text{finites}]_7$ $[\text{e}]_1$ and $[\text{-ts to unit}]_8$, where 'e' is the last letter in 'unite' and '-ts' are the last two letters of the word 'crypts'. The poem pivots around the letter-unit $[\text{key}]_3$, which corresponds to the 20th digit in the expansion.

Etherin has also written aelindromes structured around π, the golden ratio φ, and $\sqrt{2}$.

Like π and e, $\sqrt{2}$ (the square root of 2) is an irrational number. Legend attributes the discovery of irrational numbers to the Greek mathematician-philosopher Hippasus (c. 530 – c. 450 BC), who is said to have been drowned as punishment for revealing their existence. Sarah Glaz refers to this legend in the following poem. Here the number of lines per stanza correspond to the digits of the decimal expansion.

$\sqrt{2} = 1.41421 \ldots\ldots$

We started our voyage on the gulf of Tarentum.

The sea was choppy
and the brothers were restless.
At dawn, we gathered on the deck
intent to solve the conflict like rational men.

Hippasus still refused to keep the secret.

He had discovered that
the diagonal of a square
is incommensurable
with its side.

Alas! Our world had collapsed
and so did our geometric proofs.

Too much to lose, we heaved him overboard.

Using the rules of Standard Pilish, Michael Keith has written a three-stanza poem linking π, e and the golden ratio φ. The first stanza is a word-length mnemonic for

φ = 1.6180339887498948482045... (22 digits),

the second stanza is a mnemonic for

e = 2.7182818284590452353602874... (26 digits)

and the third stanza gives

π =3.1415926535897932384626433832 (29 digits).

In Pilish, each mnemonic requires exactly 134 letters, and the poem has the additional constraint that all three stanzas are exact anagrams of each other (i.e. rearrangements of the same set of 134 letters).

Pictographie

I marred a groaning silhouette,
Saw dim abhorrent freedoms cemented forever,
Till blackened paranoia bewitched this shadowed roof,
Smashing my despondent soul.

In meadows I remember my orations,
A forecast of degraded love:
Words cadential, proceeding from Hades
To the heart now shrunk,
Sublimated in helpless, binding hate.

You, a tree, a field overblown in summer,
Words and looks gathered, solicited, chanced,
Reminders now of the pleasing past
Bathed in bright heat and sad memories for me.

As well as irrational numbers and the Fibonacci sequence that we considered in Chapter 1, many other mathematical sequences have informed poetic structure. The Greek pastoral poet Theocritus (c. 300 BC - c. 260 BC) wrote a puzzle poem in which the lines are arranged in pairs, with each pair a syllable shorter than the pair before. Altogether there are 20 lines, and the solution to the puzzle is a shepherd's pipe dedicated to the god Pan.

A similar approach was adopted centuries later by the Oulipians, using the simple additive sequence 1, 2, 3, … to construct 'snowball poems'. Each line in a snowball poem consists of a single word, with each successive word one letter longer. Such poems can take various shapes: Harry Mathews' ‘Liminal Poem’, which he dedicated to the mathematics and science writer Martin Gardner, is an example of a so-called *losange* snowball, that grows and then melts.

Another mathematical sequence, formed from the coefficients of binomial expansions, is commonly presented as an array known as Pascal’s Triangle. Here are the first few rows:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The triangle is named after the 17th century French mathematician and philosopher Blaise Pascal, who described it in a paper written in 1654. In 2007 *Mathematics Magazine* published 'Dearest Blaise', a poem by Caleb Emmons using a letter count per word. Pascal's Triangle also provides the architecture for my poem 'Midwinter', which uses a letter and character count.

Midwinter

as
the day
ends sickle moon
reaps ice-clouds snowflakes drift
softly night-stillness moonlight-reflecting land-blanketing winter

Emily Galvin takes a different approach in her fine poem 'Pascal's Triangle', with the triangle's structure signified in the wording, format, and arrangement of the stanzas.

Daniel Tammet has observed that the traditional Japanese poetic forms the haiku and the tanka both use prime number sequences in their structure. The haiku, as we have already seen, has three lines of 5, 7 and 5 syllables to make a total of 17 syllables. The tanka has five lines of 5, 7, 5, 7 and 7 syllables, so 31 syllables in total. 5, 7, 17 and 31 are all prime numbers.

A striking illustration of the creative synergy that can exist between art, poetry and mathematics is artist Carl Andre's poem 'On the Sadness', which is structured according to the Fundamental Theorem of Arithmetic. The theorem states that every integer greater than 1 is either a prime number or can be expressed uniquely as a product of prime numbers. The table below illustrates the theorem for the numbers 2 through to 17:

2	prime	10	$= 2 \times 5$
3	prime	11	prime
4	$= 2^2$	12	$= 2^2 \times 3$
5	prime	13	prime
6	$= 2 \times 3$	14	$= 2 \times 7$
7	prime	15	$= 3 \times 5$
8	$= 2^3$	16	$= 2^4$
9	$= 3^2$	17	prime

In his poem, Andre assigns a phrase to each prime number, using the conjunctions 'if' and 'then' to represent the operations of multiplication and exponentiation. The first line of the poem corresponds to the prime integer 47, with the phrase 'The door is closed'. The second line, corresponding to $46 = 2 \times 23$, is 'we are going to die if the moon changes'. The complete poem can be found here. It is also included in the anthology *Strange Attractors* (2008), edited by Sarah Glaz and JoAnne Growney and listed in the Further Reading below.

Both Glaz and Growney have written poems using this structure. Glaz discusses her melodious, elegiac poem '13 January 2009' in her paper 'The Poetry of Prime Numbers' (2011). Here is Growney's 'We Are the Final Ones', which she writes about in a blog post on her website *Intersections — Poetry with Mathematics*:

We Are the Final Ones

we breathe dirty air
oil scums our waterways
we breathe dirty air as we breathe dirty air
bees disappear
we breathe dirty air and oil scums our waterways
climate change affects the poor first
we breathe dirty air as oil scums our waterways
oil scums our waterways as we breathe dirty air
we breathe dirty air and bees disappear
glaciers melt
we breathe dirty air as we breathe dirty air and oil scums our waterways
drought is a serial killer
we breathe dirty air and climate change affects the poor first
oil scums our waterways and bees disappear
we breathe dirty air as we breathe dirty air as we breathe dirty air
What will happen to the polar bears?

The poem's title corresponds to the number 1, the phrase 'we breathe dirty air' corresponds to 2 and the final line 'What will happen to the polar bears?' corresponds to 17. Multiplication is represented by 'and' and exponentiation by 'as'. Each prime number is associated with a new phrase: the numbers table above can be used to clarify the rest of the poem's structure. Note how the repetition of phrases evokes the tolling of a bell, imparting a dirge-like quality to the poem. As Sarah Glaz has observed, 'All poems I know employing this technique express feelings of sadness.'

The concepts behind the Fundamental Theorem of Arithmetic date back to Euclid, who was born in the 4^{th} Century BC and died in Alexandria. As in the case of the scribe Ahmes estimating a value for π over a millennium earlier, this illustrates how poetry and mathematics interconnect across centuries, continents, and cultures.

Further reading

Dr Tatiana Bonch-Olovskaya, (2012) 'Art of π: Mathematical History and Literary Inspiration', *Proceedings of Bridges 2012.* Available online at http://archive.bridgesmathart.org/2012/bridges2012-79.pdf

Caleb Emmons (2007) 'Dearest Blaise'. Available online at https://poetrywithmathematics.blogspot.com/2011/12/poetic-pascal-triangle.html

Anthony Etherin (2017) *Aelindromes*. Available online at https://anthonyetherin.files.wordpress.com/2017/07/aelindromes.pdf

Anthony Etherin (2019) *Stray Arts (and Other Inventions).* Penteract Press

Anthony Etherin (2024) *Knit Ink (and Other Poems).* Deep Vellum

Emily Galvin (2008) *Do the Math*. Tupelo Press, Vermont

Sarah Glaz and JoAnne Growney (eds.) (2008) *Strange Attractors: Poems of Love and Mathematics*. A.K. Peters, Massachusetts

Sarah Glaz (2010), 'The Enigmatic Number *e*: A History in Verse and Its Uses in the Mathematics Classroom'. Available at: https://www2.math.uconn.edu/~glaz/My_Articles/TheEnigmaticNumberE.Convergence10.pdf

Sarah Glaz (2011) 'The Poetry of Prime Numbers', *Proceedings of Bridges 2011*. Available online at: https://archive.bridgesmathart.org/2011/bridges2011-17.pdf

Sarah Glaz (2016) 'Poems structured by integer sequences', *Journal of Mathematics and the Arts*, 10:1-4, 44-5

Sarah Glaz (2017) *Ode to Numbers.* Antrim House, Connecticut

JoAnne Growney, *Intersections – Poetry with Mathematics.* Available online at: https://poetrywithmathematics.blogspot.com

Michael Keith, *Home Page of Michael Keith*. Available online at: http://www.cadaeic.net

Daniel May (2020) 'Poems Structured by Mathematics' in: Sriraman B. (eds) *Handbook of the Mathematics of the Arts and Sciences.* Springer, Cham.

Daniel Tammet (2012) *Thinking in Numbers*. Hodder & Stoughton