# Chapter 7: Fractals

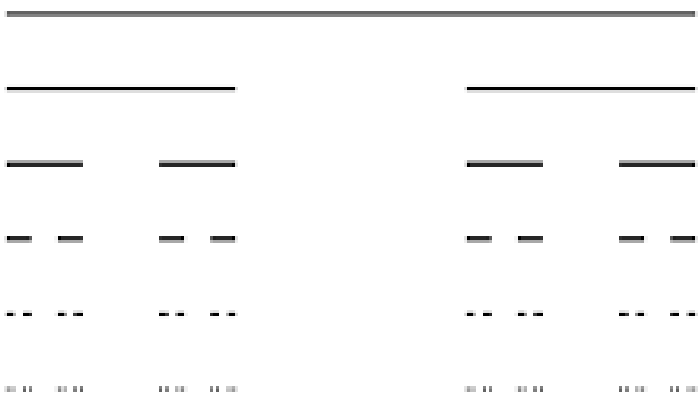
‘Clouds are not spheres’ Benoit Mandelbrot famously wrote, ‘mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.’ And perhaps we could add to this: poems are not neatly arranged blocks of text on a page – or at least, not always.

Mandelbrot made his observation in the introduction to his book *The Fractal Geometry of Nature*, in which he explored 'irregular and fragmented patterns around us' that 'tend to be *scaling*, implying that the degree of their irregularity and/or fragmentation is identical at all scales.' He called this family of shapes fractals, from the Latin adjective *fractus*, meaning fragmented or irregular.

Mandelbrot was the first to develop the field of fractal geometry and to study the occurrence of fractals in a wide range of applications. However, well before his time mathematicians had investigated some of the underlying concepts, including shapes that could not easily be described in standard geometrical terms.

One such shape is the Cantor set, which is named after the German mathematician Georg Cantor (1845 – 1918) although it was first defined in 1874 in a paper by Henry John Stephen Smith (1886 – 1883), an Irish mathematician and professor at Oxford. The most common version of the set is formed by repeatedly removing the middle third section of a straight line.

**The Cantor Set**



This set exhibits characteristics that are common to many fractals. It is formed by a simple recursive process and is 'self-similar', that is, it contains copies of itself at different scales. Furthermore, it has fine detail at every scale and cannot readily be described in terms of classical geometry. Its Hausdorff dimension (a measure of the set's irregularity) is $\ln 2/\ln 3 \approx 0.631$, a value which lies between the dimension of a point (0) and a line (1). In general, the dimensions of fractals have non-integer values.

The linear structure of the Cantor set lends itself to concrete poetry. Rodrigo Siquera's poem 'The Cantor Dust' reflects on concepts of order and chaos, referencing the set in its layout. Mike Naylor's 'Entirely Nothing', which is oriented vertically rather than horizontally, embeds the set's structure in its content as well as in its form.

**Entirely Nothing**

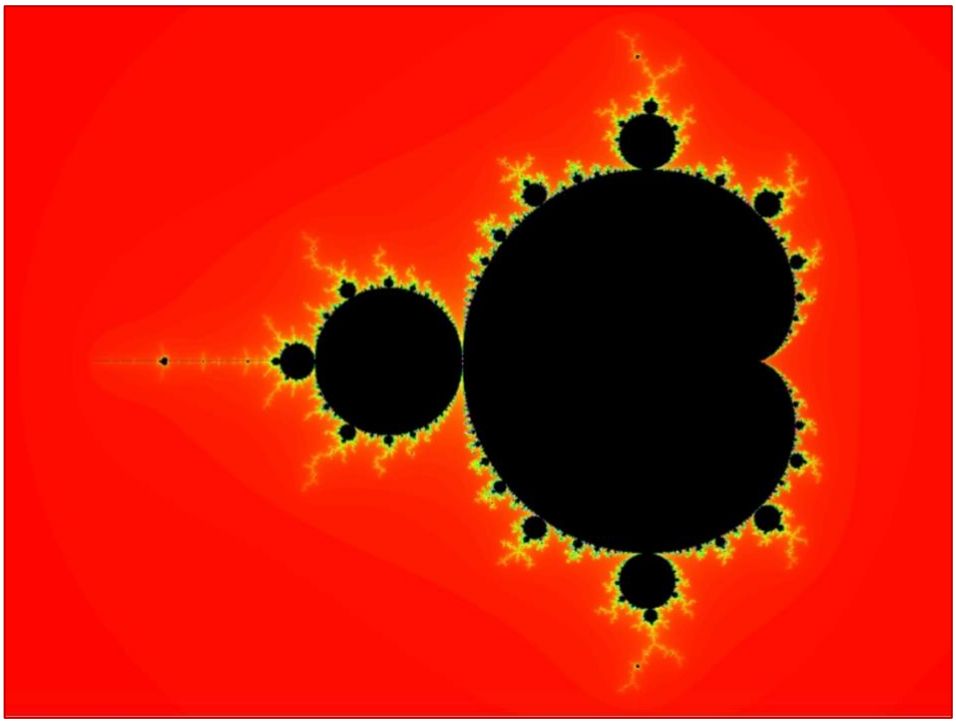
| | | | |
|---|---|---|---|
| By | By | By | By |
| keeping | keeping | keeping | |
| the | the | the | the |
| sections | sections | | |
| on | on | | |
| each | each | | |
| end | end | end | end |
| intact, | intact, | intact, | |
| all | all | all | all |
| the | | | |
| while | | | |
| removing | | | |
| the | | | |
| center | | | |
| thirds, | | | |
| eventually | | | |
| all | | | |
| you | you | you | you |
| will | will | will | |
| have | have | have | have |
| is | is | | |
| dust | dust | | |
| that | that | | |
| is | is | is | is |
| really | really | really | |
| nothing. | nothing. | nothing. | nothing. |

Another fractal, the Apollonian gasket, is named after Apollonius of Perga, whose work on conics was discussed in a previous chapter. Formed from mutually tangential circles, the Apollonian gasket provides the framework for my concrete poem below.

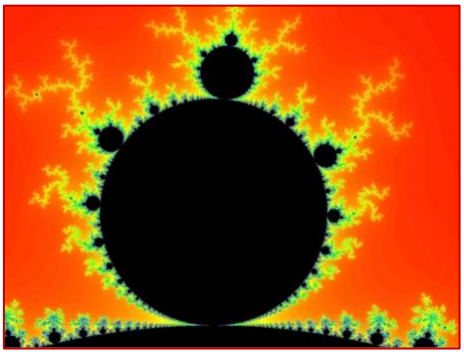
**Circles within Circles**


The most iconic mathematically generated fractal is the Mandelbrot set, which is defined by a simple recursive rule in the complex plane (the complex plane consists of all numbers that have the form $a + ib$, where $a$ and $b$ are real numbers and $i = \sqrt{-1}$). The set has an odd, dumpy shape with a 'fuzzy' boundary.

**The Mandelbrot Set**



Zooming in on the set's boundary will reveal its extraordinary beauty and intricacy in ever finer detail. The images below show a portion of the same boundary region (where the top bulb joins the main body of the set) at increasingly smaller scales.



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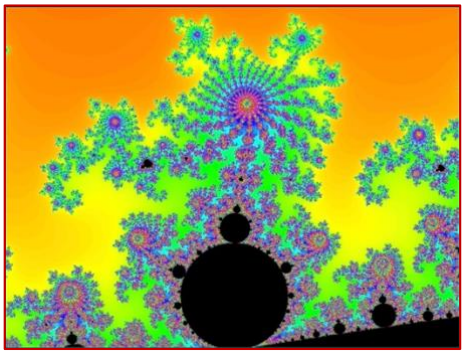

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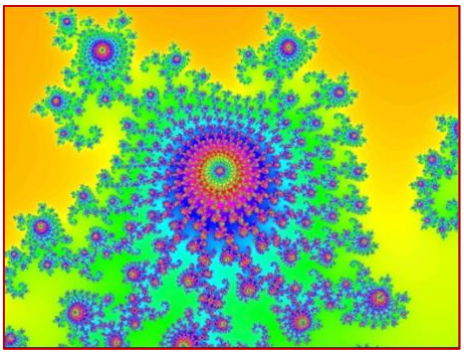

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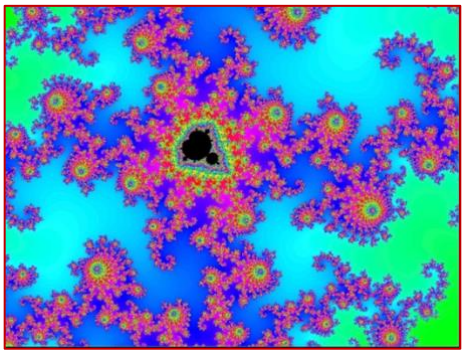

4.

The structure of Pedro Poitevin's sonnet 'Upon Inspecting the Mandelbrot Set' connotes that of the set itself. Poitevin uses metre, rhyme, half-rhyme, repetition, and alliteration, playing on words and meaning to express the set's convoluted energy, its self-similarities and surprises through shifting scales.

**Upon Inspecting the Mandelbrot Set**

I find myself absorbed by what I find
within a hole within a hole within
a surface I don't know how to begin
to trace without perceiving that my mind
is spiraling toward the undefined –
in curlicues with curlicues that spin
like pegs I turn to tune a violin
I nonetheless can't play. I stand resigned
to sink into the void, to simply sink,
to sigh because I cannot help but sigh,
to mean to see what's meant for me to see,
to think the thoughts I think, or thought I'd think,
to know the die's been cast before I die,
and be the wonder wondering to be.

In Robin Chapman's poem 'Escaping the Mandelbrot Set', the set becomes a metaphor for life's experiences.

**Escaping the Mandelbrot Set**

*Def: Equation yielding a fractal pattern self-similar at every level of magnification within a range of values.*

She says
The coffee is fine
Though it could have been stronger
And cream would be nice.

She says
The weather today
Is, yes, fine, though cold
For summer and more rain likely tonight.

She says
The summer's going well,
Of course awfully fast and won't last
Long enough to get done what she'd planned.

She says
The marriage was ten good years
And then ten bad, and she's learned
A lot since, though of course it's lonely.

She says
Buying a new cappuccino maker,
Espresso roast, and best jam for her bread
Is frivolous, but we only have one life.

Careful reading of this poem reveals fractal characteristics embedded in its structure. The refrain 'She says' recurs at the beginning of each stanza, while words or phrases repeated in successive stanzas – 'fine', 'though', 'summer', 'of course' – suggest self-similarity. We shift through time scales, rather as though we are zooming out; from the immediacy of drinking coffee in the opening stanza to the 'weather today', the summer 'going well' and then the years of marriage in the fourth stanza. The final stanza begins in the same way as the others, but there is a change in tone and vocabulary, a sense of opening out into new possibilities, of breaking free from the bounds of the metaphorical set.

Writing on what she terms ‘fractal poetics’, Alice Fulton has identified fractal elements in the work of such diverse poets as Emily Dickinson and Ezra Pound. She has described fractals as providing a framework for exploring 'the hidden structures of free verse', functioning as 'a dynamic, turbulent form between perfect chaos and perfect order'. In this context, 'Fractal poetry ... makes use of recurring cluster words, limbic lines, or canopy stanzas as a means of creating depth'. Fulton's poem 'Industrial Lace', with its clusters of words and interwoven layers of imagery, exemplifies how she has applied these concepts in her own writing.

What exactly is meant by fractal forms in poetry? Indeed, what exactly are fractals? Kenneth Falconer, Regius Professor of Mathematics at the University of St Andrews, answers the latter question as follows:

‘My personal feeling is that the definition of a 'fractal' should be regarded in the same way as a biologist regards the definition of 'life'. There is no hard and fast definition, but just a list of properties characteristic of a living thing, such as the ability to reproduce or to move or to exist to some extent independently of the

environment. Most living things have most of the characteristics on the list, though there are living objects that are exceptions to each of them.'

Characteristic properties of fractals include those we identified in our discussion of the Cantor ternary set:

- fractals are generally formed by a simple procedure, such as recursion;
- they tend to be self-similar, containing copies of themselves at different scales;
- they have detail at every scale;
- they cannot readily be described in terms of traditional geometry;
- their structure is irregular and may be fragmented.

The Cantor set, the Apollonian gasket and the Mandelbrot set are all mathematically constructed fractals, and the poems we have considered in each case signify this in their form. However, as Mandelbrot observed, fractals are present in nature: in the irregular shape of coastlines; in the branching of trees and of bronchial airways in the lungs; in fern fronds and the serrated edges of snowflakes. Analogously, fractal characteristics can arise organically in poetry, particularly perhaps in nature poetry.

As an example, let's consider 'Driftwood – Olympic Peninsula', by American poet Devon Marsh.

**Driftwood – Olympic Peninsula**

Children throw driftwood into the sea,
rebuking the wood's audacity.

Limbs and trunks lie ashore, bone-like
rather than a severed part of forest. Yet

the skeleton-white logs pay homage
to shadow-black forms. Trees above wave-carved

cliffs oversee the beach. Tall firs look ahead.
The firs turn from the ocean, put breath

into the sky. They look back and think
again. Children cast pieces of trees

to withdraw with the tide and feign they won't
return when we're gone. Tall firs regard the sea.

They look back and think of me, know
I am of the forest even as rhythmic waves

pound within my veins. How many
will pulse the shore where I stand

rooted to a spot on the sand? I take evergreen
breath from the sky, look back, breathe again.

I watch my heirs, relish their audacity.
My children throw driftwood into the sea.

This poem was not consciously written with fractals in mind; yet it contains fractal elements in both its content and its structure. The coastal setting, the 'wave-carved cliffs' and the 'tall firs' all attest to the presence of fractals in nature. The trees are fragmented – 'Limbs and trunks lie ashore'. There is a sense of scaling: the parent and the children; the forest, the trees and the driftwood; tides, waves, heartbeats. Recursion is signified by the driftwood that is cast out to sea and returns with the incoming tide.

Structurally, the use of repeated words, phrases and images suggests self-similarity. 'Trees', 'tall firs', 'sea', 'they look back and think', 'breath', 'sky', 'audacity' all occur more than once in the poem. The 'I'-persona stands 'rooted to a spot on the sand' just as the trees are rooted to the cliffs.

The recursive elements throughout this poem also contain a subtle positive dynamic: in Marsh's words, 'the hope of linear progress, as with descendants who inherit our troubled world and may be able to heal it.'

**Further reading**

Marcia Birken and Anne C. Coon (2008) *Discovering Patterns in Mathematics and Poetry*. Editions Rodopi

Robin Chapman and Julien Clinton Sprott (2005) *Images of a Complex World: The Art and Poetry of Chaos*. World Scientific Publications

Marian Christie (2021) *Fractal Poems*. Enneract Editions, Penteract Press

Kenneth Falconer (2014) *Fractal Geometry: Mathematical Foundations and Applications* (third edition). Wiley

Alice Fulton (1995) 'Industrial Lace' from *Sensual Math*, Norton. Available at: https://www.poetryfoundation.org/poems/47806/industrial-lace

Alice Fulton (1998) 'Fractal Amplifications: Writing in Three Dimensions' in *Thumbscrew No. 12 - Winter 1998/9*. Available at: http://poetrymagazines.org.uk/magazine/recordd738.html?id=12199

Alice Fulton (2005) 'Fractal Poetics: Adaptation and Complexity' in *Interdiscplinary Science Review*. Available at: https://alicefulton.com/books/isr2005.html

Sarah Glaz (2011) 'Poetry Inspired by Mathematics: a Brief Journey through History'. Available at: https://studylib.net/doc/8786715/poetry-inspired-by-mathematics--a-brief-journey-through-history

Benoit B. Mandelbrot (1983) *The Fractal Geometry of Nature*. Freeman

Mandelbrot Viewer. *Explore the Mandelbrot Set.* Available at: https://math.hws.edu/eck/js/mandelbrot/MB.html

Devon Marsh, *iamb—poetry seen and heard, 2021*. Available at: https://www.iambapoet.com/wave/7-devon-marsh

Pedro Poitevin (2023) *Nowhere at Home*. Penteract Press

Rodrigo Siqueras, 'The Cantor Dust'. Available at http://www.insite.com.br/rodrigo/misc/fractal/fractal_poetry.html